

TAKEAWAY TASKS NO 2

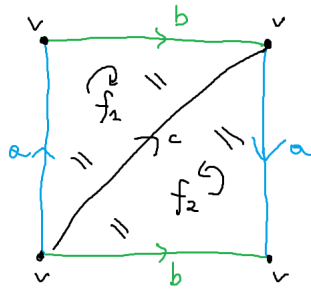
(1) Compute the homology of the simplicial complex K whose maximal faces are:

$$\{[1,2],[1,3],[2,3],[1,4,5],[1,4,6],[1,5,6],[4,5,6]\}.$$

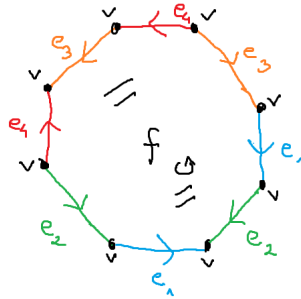
(2) Compute the homology of the simplicial complex K whose maximal faces are:

$$\{[1,2],[2,3],[1,5],[2,5],[1,3,4]\}.$$

(3) Compute the homology of the *Klein bottle* using its Δ -complex structure depicted in the picture below.



(4) Compute the homology of the *orientable surface of genus 2* using its *cell* complex structure depicted in the picture below.



(5) Prove that the homology of a connected graph G without cycles consisting of finitely many vertices (and no multiple edges between the same two vertices allowed) is as follows:

$$H_n(G) = \begin{cases} \mathbb{Z} & n = 0, \\ 0 & n \neq 0. \end{cases}$$